5.1/11 $y = x^2 - 5x + 6$

odd
even

$y' = 2x - 5$

$= 0 = 2x - 5$

$x = \frac{5}{2}$

s.c. g f'

$f'- \Rightarrow f$ dec $\quad f'+ \Rightarrow f$ inc

$- \; - \; - \quad + \; + \; +$

$2x-5<0 \qquad \frac{5}{2} \qquad 2x-5>0$

$2x5=0$

$f'(x) > 0$

$\Rightarrow f^n \, f(x)$ inc

$f'(x) < 0$

$\Rightarrow f(x)$ dec

$y' = 2x - 5$

$y'' = 2$

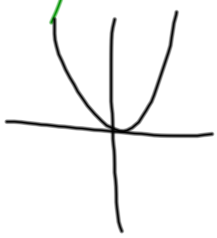
$f''+ \Rightarrow f$ is concave up

$f''- \Rightarrow f$ is concave down

$+ \; + \; + \; + \; + \; +$

1) sketch a graph of
an INCREASING function

2) sketch a graph of a function
with an INCREASING
first derivative



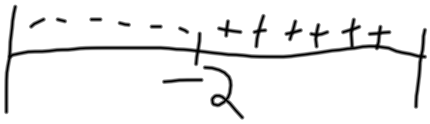
3) sketch a graph with
an INCREASING
second derivative

5.1/13

$$y=(x+2)^3$$

$$f'(x)=3(x+2)^2$$

$$f''(x)=6(x+2)$$



inc: $(-\infty,\infty)$

dec: (never, never) null set, never $\emptyset$

c. up: $(-2,\infty)$

c. dn: $(-\infty,-2)$

pt of inflection: (-2) $\{-2\}$

5.1

17) $y = \frac{x^2}{x^2+2}$

$$y' = \frac{(2x)(x^2+2) - (x^2)(2x)}{(x^2+2)^2}$$

$$\frac{2x^3+4x-2x^3}{(x^2+2)^2} \qquad y' = \frac{4x}{(x^2+2)^2}$$

$- - - - \mid + + + +$ (sign chart, critical point at 0)

a) increasing $= [0, +\infty)$

b) dec. $= (-\infty, 0]$

$$y' = \frac{f'g - fg'}{g^2}$$

$$y'' \quad \frac{(4)((x^2+2)^2) - (4x)(4x(x^2+2))}{(x^2+2)^4}$$

Sign chart
- solve num $= 0$
- solve den $= 0$

$\cancel{\varnothing}$

$4(x^2+2)^2 - 16x^2(x^2+2)$

$4(x^2+2)[x^2+2-4x^2]$

$$\frac{4(x^2+2)[2-3x^2]}{}$$

$2-3x^2=0$

$3x^2=2 \quad x^2=\frac{2}{3} \quad x=\pm\sqrt{\frac{2}{3}}$

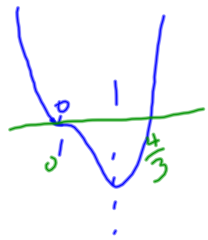
s.c of f''

$- - - \mid + + + \mid - - -$

$-10 \quad -\sqrt{\frac{2}{3}} \quad 0 \quad +\sqrt{\frac{2}{3}} \quad 10$

5-Y (15)

$$y = 3x^4 - 4x^3 = x^3(3x-4)$$



$$y' = 12x^3 - 12x^2 = 0$$

$$12x^2(x-1) = 0$$

$$x = 0, 1$$

sign chart of y'

- solve for num = 0
- solve for den = 0

* plot those pts on a # line

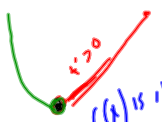
* check 1 pt in each interval

$12x^2(x-1) = 0$

$- - - - - \mid - - - \mid + + +$

$-1 \quad 0 \quad \frac{1}{2} \quad 1 \quad 9999$

$y' = 0 \qquad y' = 0$



$f(x)$ is increasing on an interval if $f(x) > f(y)$ whenever $x > y$

$f(x)$ is INCREASING whenever $f'(x) > 0$ i.e. $[1, \infty)$

$f(x)$ is DECREASING whenever $f'(x) < 0$ i.e. $(-\infty, 1]$

$$y' = 12x^3 - 12x^2$$

$$y'' = 36x^2 - 24x = 12x(3x-2)$$

$$x = 0, \tfrac{2}{3}$$

sign chart

$+ + + \mid - - - \mid + + +$

$-1 \quad 0 \quad \frac{1}{2} \quad \frac{2}{3} \quad 9999$

$\therefore$ $f(x)$ is concave up when $f''(x) > 0$

$f(x)$ is concave down when $f''(x) < 0$

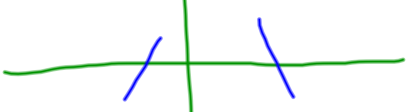
Pt of inflection: is a PT where concavity chgs:

$x = 0, x = \frac{2}{3}$

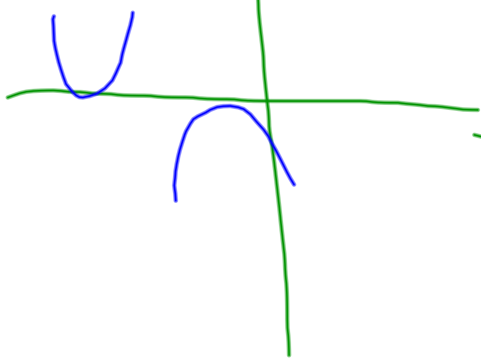
Segues - - - - Multiplicities

mult of 1

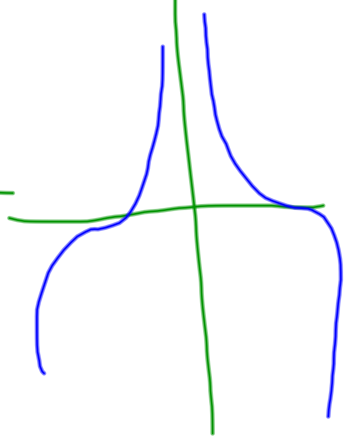
$P(x) = (x+1)(\quad)$



even multiplicity



odd multiplicities



multiplicities of factors

multiplicities of root

17) $y = \dfrac{x^2}{x^2+2}$

$$y' = \frac{2x(x^2+2) - (x^2)(2x)}{(x^2+2)^2}$$

inc: $[0, \infty)$

dec: $(-\infty, 0]$

$$= \frac{4x}{(x^2+2)^2}$$

$- - - - \,|\, + + + +$

0

$4x = 0$

$\Rightarrow x = 0$

$$y' = \frac{4x}{(x^2+2)^2}$$

$$y'' = \frac{(4)(x^2+2)^2 - 4x\left(2(x^2+2)2x\right)}{(x^2+2)^4}$$

$\boxed{\begin{array}{l} a(b+c) \\ = ab + ac \end{array}}$

$$= \frac{4(x^2+2)\left[(x^2+2) - 4x^2\right]}{(x^2+2)^4}$$

$$= \frac{4(x^2+2)\left[2 - 3x^2\right]}{(x^2+2)^4}$$

$2 - 3x^2 = 0$

$2 = 3x^2$

$\frac{2}{3} = x^2$

$x = \pm\sqrt{\frac{2}{3}}$

sign $\dfrac{4(x^2+2)(2-3x^2)}{(x^2+2)^4}$

$- - - - - \,|\, + + + + \,|\, - - -$

$-1 \quad -\sqrt{\frac{2}{3}} \quad 0 \quad \sqrt{\frac{2}{3}} \quad 9999$

$0 \qquad 0$

c-up: $\left(-\sqrt{\frac{2}{3}}, +\sqrt{\frac{2}{3}}\right)$

c-dn: $\left(-\infty, -\sqrt{\frac{2}{3}}\right) \cup \left(+\sqrt{\frac{2}{3}}, \infty\right)$

pt of inf: $-\sqrt{\frac{2}{3}}, +\sqrt{\frac{2}{3}}$ (x-values)